Name ______________________ Date ______________________ *AP Calculus AB/BC*

1. Let f be the function defined by $f(x) = x\cos x + 1$, where x is measured in radians. Consider the four intervals $[0, 1]$, $[1, 2]$, $[2, 3]$, and $[3, 4]$. On how many of these four intervals does the Intermediate Value Theorem guarantee that f has at least one zero?

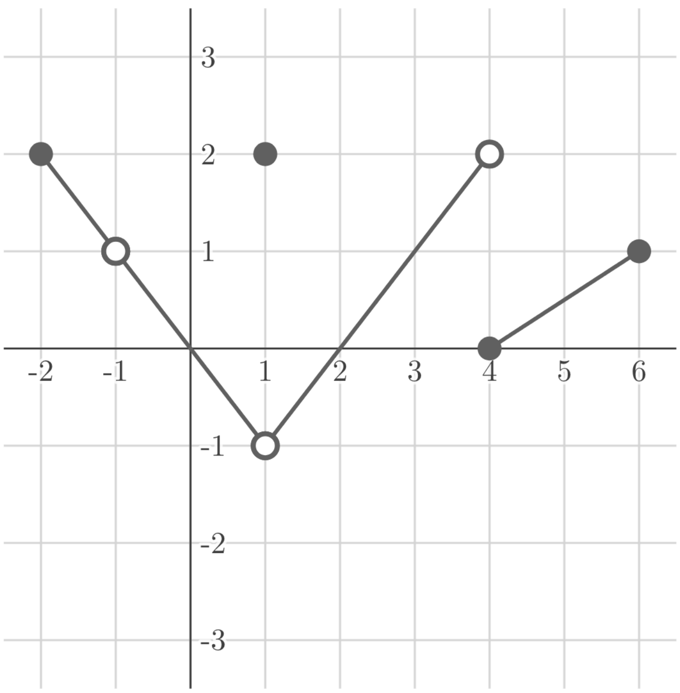
   A) 0

   B) 1

   C) 2

   D) 3

2. The graph of the function f is shown for $-2 \le x \le 6$. At which values of c in the open interval $(-2,\ 6)$ does $\lim_{x\to c} f(x)$ exist while f is NOT continuous at c?



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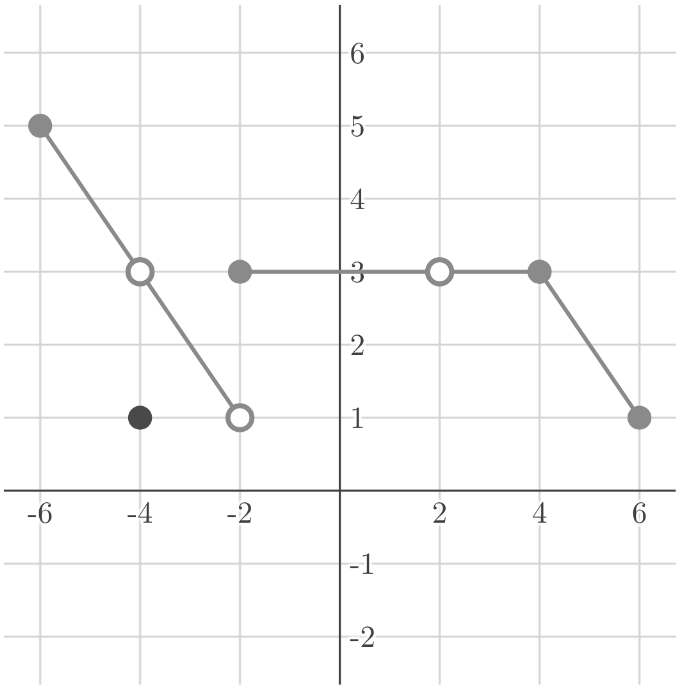
3. The graph of the function g is shown for $-6 \leq x \leq 6$. For how many values of c with $-6 < c < 6$ does $\lim_{x \to c} g(x)$ exist while g fails to be continuous at c?



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4. The function f is defined for $x > -2$ and $x \neq 2$ by

$$f(x) = \frac{x^2 + x - 6}{\sqrt{x+2} - 2}.$$

What value must be assigned to $f(2)$ so that f is continuous at $x = 2$?

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5. The function f is defined by

$$f(x) = \begin{cases} ax + b, & x < 1 \\ x^2 + 4, & 1 \le x < 4 \\ ax^2 - bx, & x \ge 4 \end{cases}$$

where a and b are constants. If f is continuous for all real numbers x, what is the value of $a + 2b$?

A) 5

B) 7

C) 8

D) 10

6. Let f be the function defined by

$$f(x) = \frac{x^3 - 4x}{x^3 - x^2 - 6x}.$$

Which of the following statements about the discontinuities of f is true?

A) f has one removable discontinuity and two vertical asymptotes.

B) f has two removable discontinuities and one vertical asymptote.

C) f has no removable discontinuities and three vertical asymptotes.

D) f is discontinuous only at $x = 3$, because the factors that cancel remove the other discontinuities entirely.

7. Let $g(x) = x \sin x$ and let $f(u) = \frac{u-2}{u-1}$, where x is measured in radians. For how many values of x in the open interval $0 < x < 6$ is the composite function $f(g(x))$ discontinuous?

A) 0

B) 1

C) 2

D) 3

8. A shipping service charges 6 dollars for a package weighing more than 0 pounds and at most 1 pound, and each additional pound or part of a pound adds 2 dollars. Let $C(w)$ be the cost, in dollars, of shipping a package weighing w pounds, for $0 < w \leq 6$. Which of the following statements about C at $w = 3$ is true?

   A) $\lim_{w\to 3} C(w) = 10$ and $C(3) = 10$, so C is continuous at $w = 3$.

   B) C is continuous from the left at $w = 3$, but C is not continuous at $w = 3$.

   C) C is continuous from the right at $w = 3$, but C is not continuous at $w = 3$.

   D) C has a removable discontinuity at $w = 3$, since $\lim_{w\to 3} C(w)$ exists but differs from $C(3)$.

9. Let $f(x) = \frac{3^x - 3^{-x}}{x}$ for $x \neq 0$. Use a calculator to evaluate f at values of x very close to 0. Which of the following is the best estimate of $\lim_{x\to 0} f(x)$?

   A) 0

   B) 1.099

   C) 2.197

   D) 3

10. $$f(x) = \begin{cases} 2x + a, & x < -1 \\ x^2 + bx, & -1 \leq x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$$

   The function f is defined above, where a and b are constants. If f is continuous for all real numbers, what are the values of a and b?

   A) $a = 2,\ b = 1$

   B) $a = 4,\ b = 1$

   C) $a = 1,\ b = 2$

   D) $a = 5,\ b = 2$

11. Let $f(x) = \frac{x^3 - 9x}{x^2 + x - 6}$. What is $\lim_{x\to 2^-} f(x)$?

   A) $-\infty$

   B) 0

   C) -2

   D) ∞

12. Let

$$f(x) = \frac{x^2 + ax - 12}{x - 3}, \qquad x \neq 3,$$

where a is a constant. There is exactly one value of a for which $\lim_{x\to 3} f(x)$ is a finite number. Find that value of a, and let L be the resulting value of the limit.

a: ______________________________

L: ______________________________

13. Evaluate $\lim_{x\to 0} \frac{1-\cos(4x)}{x\sin(3x)}$.

A) 0

B) $\frac{8}{3}$

C) $\frac{9}{8}$

D) $\frac{16}{3}$

14. The function f satisfies

$$6x^2 - x^3 \;\le\; x^2 f(x) \;\le\; 6x^2 + 4x^4$$

for all $x \neq 0$. Find $\lim_{x\to 0} f(x)$.

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15. The function f is defined by $f(x) = \frac{\sqrt{x+7}-3}{x-2}$ for all $x \neq 2$. A value $f(2) = c$ is assigned so that the extended function is continuous at $x = 2$. Enter the value of c as a fraction.

c: ______________________________

16. The function f is defined by $f(x) = e^{x/2} - x^2$ and is continuous on $[-2, 4]$. For which of the following intervals does the Intermediate Value Theorem guarantee that $f(x) = 0$ for some x in that interval?

   A) $[-2, -1]$
   B) $[0, 1]$
   C) $[1, 2]$
   D) $[2, 3]$

17. Let f be the function defined by

$$f(x) = \frac{3^x - 3^{-x}}{x}, \qquad x \neq 0.$$

   Using a calculator to evaluate f at values of x very close to 0, which of the following is the best approximation of $\lim_{x\to 0} f(x)$?

   A) 0
   B) 0.954
   C) 1.099
   D) 2.197

18. Let g be the function defined by

$$g(x) = \frac{x^2 + kx - 6}{x^2 - 9},$$

   where k is a constant. There is exactly one value of k for which $\lim_{x\to 3} g(x)$ exists. For that value of k, find $\lim_{x\to 3} g(x)$. Enter your answer as a fraction in lowest terms.

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19. The function f is defined by

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x < 2 \\ b \sin x, & x \geq 2 \end{cases}$$

where b is a constant and x is measured in radians. Which value of b makes f continuous at $x = 2$?

A) 3.637

B) 4.000

C) 4.399

D) 114.615

20. The function f is continuous on the closed interval $[0, 12]$. Selected values of f are given in the table.

x	0	2	5	7	9	12
$f(x)$	-4	3	1	6	2	-3

What is the least number of solutions of the equation $f(x) = 2$ that must lie in the interval $[0, 12]$?

A) 2

B) 3

C) 4

D) 5

21. Evaluate the limit.

$$\lim_{x \to -3} \frac{\sqrt{x^2 + 7} - 4}{x^2 + 4x + 3}$$

A) $-\dfrac{3}{4}$

B) $-\dfrac{3}{8}$

C) 0

D) $\dfrac{3}{8}$

22. The function f satisfies

$$4 - x^2 \ \leq \ f(x) \ \leq \ 4 + x^2$$

for all real numbers x. What is $\lim_{x \to 0} \left(f(x) \cdot \dfrac{\sin 3x}{x} \right)$?

A) 0

B) $\dfrac{4}{3}$

C) 4

D) 12

Chapter 1 · AP Review — Answer Key

Answer key — not for distribution *AP Calculus AB/BC*

1. **Answer:** B. 1

   $f(x) = x\cos x + 1$ is built from functions that are continuous everywhere, so the Intermediate Value Theorem applies on all four intervals. What it needs in order to guarantee a zero is a sign change between the endpoint values. Evaluate f at the five endpoints, in radian mode:

$$f(0) = 0 + 1 = 1$$

$$f(1) = 1\cos 1 + 1 \approx 1.540$$

$$f(2) = 2\cos 2 + 1 \approx 0.168$$

$$f(3) = 3\cos 3 + 1 \approx -1.970$$

$$f(4) = 4\cos 4 + 1 \approx -1.615$$

   Interval by interval: $[0, 1]$: positive at both ends, no sign change. $[1, 2]$: positive at both ends, no sign change. $[2, 3]$: $f(2) \approx 0.168 > 0$ and $f(3) \approx -1.970 < 0$. Since 0 lies between these values, the theorem guarantees a c in $(2, 3)$ with $f(c) = 0$. $[3, 4]$: negative at both ends, no sign change. One interval qualifies, so the answer is 1. A calculator left in degree mode gives $f(1) \approx 2.000$, $f(2) \approx 2.999$, $f(3) \approx 3.996$, and $f(4) \approx 4.997$, all positive, and produces a count of 0. Keep in mind as well that the absence of a sign change does not prove there is no zero on an interval; it only means the theorem promises nothing there.

2. **Answer:** B. $c = -1$ and $c = 1$

   Check each place where the graph breaks. At $c = -1$: the graph approaches the hollow point $(-1,\ 1)$ from both sides, so $\lim_{x\to-1} f(x) = 1$. There is no solid point at $x = -1$, so $f(-1)$ is undefined and f is not continuous there. The limit exists and continuity fails, so this value qualifies. At $c = 1$: from the left the graph approaches $(1,\ -1)$, and from the right it also approaches $(1,\ -1)$, so $\lim_{x\to1} f(x) = -1$. The solid point sits at $(1,\ 2)$, so $f(1) = 2 \neq -1$. Again the limit exists while continuity fails, so this value qualifies. At $c = 4$: from the left the graph approaches 2 (hollow point at $(4,\ 2)$) and from the right it starts at 0 (solid point at $(4,\ 0)$). The one-sided limits are 2 and 0. They disagree, so $\lim_{x\to4} f(x)$ does not exist and this value fails the first requirement. Everywhere else the graph is an unbroken line, so f is continuous. The answer is $c = -1$ and $c = 1$.

3. **Answer:** B. 2

Work left to right and stop wherever the graph is interrupted. $x = -4$: the slanted branch runs into an open circle at $(-4, 3)$ from the left and leaves an open circle at $(-4, 3)$ on the right, so $\lim_{x \to -4} g(x) = 3$. The solid dot sits at $(-4, 1)$, so $g(-4) = 1$. The limit exists and misses the function value, so g is discontinuous here and $c = -4$ qualifies. $x = -2$: from the left the slanted branch approaches $-(-2) - 1 = 1$, while from the right the horizontal branch starts at height 3. The one-sided limits disagree, so $\lim_{x \to -2} g(x)$ does not exist. This one fails. $x = 2$: the horizontal segment approaches height 3 from both sides, so $\lim_{x \to 2} g(x) = 3$. The open circle at $(2, 3)$ has no solid dot anywhere to replace it, so $g(2)$ is undefined and g is not continuous at 2. The limit exists, so $c = 2$ qualifies. $x = 4$: the horizontal segment ends at the solid point $(4, 3)$ and the falling segment begins at that same solid point, so $\lim_{x \to 4} g(x) = 3 = g(4)$. There is a sharp corner here, but a corner is not a discontinuity, so this one fails. Two values qualify: $c = -4$ and $c = 2$.

4. **Answer:** 20

Continuity at $x = 2$ requires $f(2) = \lim_{x \to 2} f(x)$, so everything comes down to that limit. Substituting gives $\frac{4+2-6}{\sqrt{4}-2} = \frac{0}{0}$, an indeterminate form. Clear the radical from the denominator by multiplying numerator and denominator by the conjugate $\sqrt{x+2} + 2$:

$$f(x) = \frac{(x^2 + x - 6)\left(\sqrt{x+2} + 2\right)}{\left(\sqrt{x+2} - 2\right)\left(\sqrt{x+2} + 2\right)} = \frac{(x^2 + x - 6)\left(\sqrt{x+2} + 2\right)}{(x+2) - 4} = \frac{(x^2 + x - 6)\left(\sqrt{x+2} + 2\right)}{x - 2}.$$

Now factor the numerator: $x^2 + x - 6 = (x+3)(x-2)$. The factor $x - 2$ cancels, so for every x in the domain with $x \neq 2$,

$$f(x) = (x+3)\left(\sqrt{x+2} + 2\right).$$

This last expression is continuous at $x = 2$, so

$$\lim_{x \to 2} f(x) = (2+3)\left(\sqrt{4} + 2\right) = 5 \cdot 4 = 20.$$

Assigning $f(2) = 20$ removes the discontinuity.

5. **Answer:** C. 8

Every branch is a polynomial, so the only places continuity can fail are the two junctions $x = 1$ and $x = 4$. At each one the left-hand limit, the right-hand limit, and the function value have to agree. At $x = 1$: the left-hand limit is $a(1) + b = a + b$, and $f(1) = 1^2 + 4 = 5$. Continuity requires

$$a + b = 5.$$

At $x = 4$: the left-hand limit is $4^2 + 4 = 20$, and $f(4) = a(4)^2 - b(4) = 16a - 4b$. Continuity requires

$$16a - 4b = 20, \quad \text{that is,} \quad 4a - b = 5.$$

Add the two equations: $5a = 10$, so $a = 2$, and then $b = 5 - 2 = 3$. Check the second junction: $16(2) - 4(3) = 32 - 12 = 20$, matching the middle branch. Therefore $a + 2b = 2 + 2(3) = 8$.

6. **Answer:** B. f has two removable discontinuities and one vertical asymptote.

   Factor the top and the bottom completely. Numerator: $x^3 - 4x = x(x^2 - 4) = x(x-2)(x+2)$. Denominator: $x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x-3)(x+2)$. The denominator is zero at $x = 0$, $x = -2$, and $x = 3$, so those are the only three candidates for discontinuity. Cancelling the shared factors x and $x + 2$ gives, for $x \notin \{0, -2, 3\}$,

   $$f(x) = \frac{x-2}{x-3}.$$

   At $x = 0$: the factor cancelled, so the limit is finite, $\lim_{x\to 0} f(x) = \frac{-2}{-3} = \frac{2}{3}$, while $f(0)$ is undefined. That is a removable discontinuity, a hole. At $x = -2$: the factor cancelled here too, so $\lim_{x\to -2} f(x) = \frac{-4}{-5} = \frac{4}{5}$, while $f(-2)$ is undefined. A second removable discontinuity. At $x = 3$: the factor $x - 3$ remains in the denominator and the numerator $x - 2$ equals 1 there, so $|f(x)| \to \infty$. That one is nonremovable, a vertical asymptote. So f has two removable discontinuities and one vertical asymptote. Cancelling a factor simplifies the formula without restoring the missing points to the domain, which is why the holes at $x = 0$ and $x = -2$ still count as discontinuities.

7. **Answer:** C. 2

   Write the composite as a single expression:

   $$f(g(x)) = \frac{x\sin x - 2}{x\sin x - 1}.$$

   Both the numerator and the denominator are continuous everywhere, so the quotient is continuous except where the denominator is zero. The discontinuities are the solutions of

   $$x\sin x = 1 \qquad \text{on } 0 < x < 6.$$

   Graph $y = x\sin x$ together with the horizontal line $y = 1$ on $0 < x < 6$, in radian mode. The curve rises from 0, crosses $y = 1$ near $x \approx 1.114$, peaks at about 1.820 near $x \approx 2.029$, and falls back through $y = 1$ near $x \approx 2.773$. For $\pi < x < 6$ the factor $\sin x$ is negative, so $x\sin x < 0$ and the curve never returns to the level 1. That gives two solutions. Check that neither one is cancelled by the numerator: at those inputs $x\sin x = 1$, so the numerator equals $1 - 2 = -1 \neq 0$. Both are nonremovable discontinuities, and the answer is 2. The equation $x\sin x = 2$ has no solution on this interval, since the maximum of $x\sin x$ here is about 1.820, so the numerator never vanishes and creates no extra cases.

8. **Answer:** B. C is continuous from the left at $w = 3$, but C is not continuous at $w = 3$.

   Write out the cost, watching which endpoint each weight interval includes. A package of exactly 1 pound still costs the base rate, and anything heavier moves up a step, so

   $$C(w) = 6 \text{ on } (0,1], \quad 8 \text{ on } (1,2], \quad 10 \text{ on } (2,3], \quad 12 \text{ on } (3,4], \quad 14 \text{ on } (4,5], \quad 16 \text{ on } (5,6].$$

   Now work at $w = 3$. Value: 3 belongs to the interval $(2,3]$, so $C(3) = 10$. Left-hand limit: weights slightly below 3 also lie in $(2,3]$, so $\lim_{w\to 3^-} C(w) = 10$. Right-hand limit: weights slightly above 3 lie in $(3,4]$, so $\lim_{w\to 3^+} C(w) = 12$. The one-sided limits are 10 and 12. They disagree, so $\lim_{w\to 3} C(w)$ does not exist and C is not continuous at $w = 3$. The left-hand limit does equal $C(3)$, however, so C is continuous from the left at $w = 3$. The discontinuity here is a jump. A removable discontinuity would require the two-sided limit to exist, and it does not.

9. **Answer:** C. 2.197

   At $x = 0$ the quotient reads $\frac{1-1}{0} = \frac{0}{0}$, so substitution settles nothing. Build a small table with a calculator: $f(0.01) = \frac{3^{0.01} - 3^{-0.01}}{0.01} \approx \frac{1.011047 - 0.989074}{0.01} \approx 2.1973$ $f(0.001) \approx \frac{1.0010992 - 0.9989020}{0.001} \approx 2.1972$ Replacing x by $-x$ negates the numerator and the denominator both, so f is even and $f(-0.001) \approx 2.1972$ as well. The values settle near 2.197 from both sides, so $\lim_{x \to 0} f(x) \approx 2.197$. (The exact value is $2 \ln 3 \approx 2.1972$.)

10. **Answer:** A. $a = 2,\ b = 1$

    Continuity can only fail where the definition changes, at $x = -1$ and $x = 2$; on each piece f is a polynomial. At a seam the two one-sided limits have to agree with the defined value. Start at $x = 2$, where only one unknown appears. The middle piece gives $f(2) = 2^2 + b(2) = 4 + 2b$, and the right piece gives $\lim_{x \to 2^+} f(x) = 5(2) - 4 = 6$. Setting $4 + 2b = 6$ gives $b = 1$. Now use $x = -1$. The left piece gives $\lim_{x \to -1^-} f(x) = 2(-1) + a = a - 2$. The middle piece gives $f(-1) = (-1)^2 + b(-1) = 1 - b = 1 - 1 = 0$. Setting $a - 2 = 0$ gives $a = 2$. Check: with $a = 2$ and $b = 1$, at $x = -1$ both sides give 0, and at $x = 2$ both sides give 6. So $a = 2$ and $b = 1$.

11. **Answer:** D. ∞

    Factor top and bottom: $x^3 - 9x = x(x-3)(x+3)$ and $x^2 + x - 6 = (x+3)(x-2)$. Cancelling the common factor $x + 3$ (valid for $x \neq -3$, which does not affect behavior near $x = 2$) gives

    $$f(x) = \frac{x(x-3)}{x-2}.$$

    At $x = 2$ the numerator equals $2(2-3) = -2$, a nonzero number, and the denominator equals 0. The value blows up rather than settling, and the sign decides which way. Approaching from the left, $x < 2$, so $x - 2$ is a small negative number. A negative numerator divided by a small negative denominator is a large positive number:

    $$\lim_{x \to 2^-} f(x) = \frac{-2}{0^-} = \infty.$$

    Numerically: $f(1.9) = \frac{1.9(-1.1)}{-0.1} \approx 20.9$, and $f(1.99) \approx 200.9$.

12. **Answer:** a: 1, L: 7

    The denominator approaches 0 as $x \to 3$. For the quotient to have a finite limit, the numerator has to approach 0 as well; any other value would send the quotient off to infinity. So

    $$3^2 + a(3) - 12 = 0 \implies 9 + 3a - 12 = 0 \implies a = 1.$$

    With $a = 1$ the numerator is $x^2 + x - 12 = (x+4)(x-3)$, so for $x \neq 3$

    $$f(x) = \frac{(x+4)(x-3)}{x-3} = x + 4.$$

    Therefore $L = \lim_{x \to 3} f(x) = 3 + 4 = 7$. For any other a the numerator at $x = 3$ equals $3a - 3 \neq 0$ and the quotient is unbounded near 3, so $a = 1$ is the only value that works.

13. **Answer:** B. $\dfrac{8}{3}$

The quotient is $\frac{0}{0}$ at $x = 0$, so rewrite it. Multiply numerator and denominator by $1 + \cos(4x)$:

$$\frac{1-\cos(4x)}{x\sin(3x)} \cdot \frac{1+\cos(4x)}{1+\cos(4x)} = \frac{1-\cos^2(4x)}{x\sin(3x)\,(1+\cos(4x))} = \frac{\sin^2(4x)}{x\sin(3x)\,(1+\cos(4x))}.$$

Now split into pieces whose limits are known, pairing each sine with a matching linear factor:

$$\frac{\sin(4x)}{4x} \cdot \frac{\sin(4x)}{4x} \cdot \frac{3x}{\sin(3x)} \cdot \frac{16x^2}{3x^2} \cdot \frac{1}{1+\cos(4x)}.$$

As $x \to 0$, the first three factors each approach 1, the fourth is the constant $\frac{16}{3}$, and the last approaches $\frac{1}{1+1} = \frac{1}{2}$. Therefore the limit is $\frac{16}{3} \cdot \frac{1}{2} = \frac{8}{3}$. As a check, at $x = 0.01$: $\frac{1-\cos(0.04)}{0.01\sin(0.03)} \approx \frac{0.00079989}{0.00029996} \approx 2.667 = \frac{8}{3}$.

14. **Answer:** 6

The inequality bounds $x^2 f(x)$, not $f(x)$, so isolate $f(x)$ first. For every $x \neq 0$ the quantity x^2 is strictly positive, so dividing all three parts by x^2 leaves the inequality signs unchanged:

$$6 - x \;\le\; f(x) \;\le\; 6 + 4x^2 \qquad \text{for all } x \neq 0.$$

Now take limits of the two outer expressions:

$$\lim_{x\to 0}(6 - x) = 6, \qquad \lim_{x\to 0}(6 + 4x^2) = 6.$$

Since the two bounds share the common limit 6 and f is trapped between them near $x = 0$, the Squeeze Theorem gives $\lim_{x\to 0} f(x) = 6$. Left alone, the original inequality only gives $\lim_{x\to 0} x^2 f(x) = 0$, since both of its outer bounds go to 0. That says nothing about f on its own, which is why the division by x^2 comes first.

15. **Answer:** $1/6$

Continuity at $x = 2$ requires $c = f(2) = \lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2}$, and putting $x = 2$ in gives $\frac{3-3}{0} = \frac{0}{0}$, so the expression has to be rewritten. Multiply the numerator and denominator by the conjugate $\sqrt{x+7}+3$:

$$\frac{\sqrt{x+7}-3}{x-2} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} = \frac{(x+7)-9}{(x-2)\left(\sqrt{x+7}+3\right)} = \frac{x-2}{(x-2)\left(\sqrt{x+7}+3\right)}.$$

For $x \neq 2$ the factor $x - 2$ cancels, leaving $\frac{1}{\sqrt{x+7}+3}$, which is continuous at $x = 2$. So

$$\lim_{x\to 2} f(x) = \frac{1}{\sqrt{9}+3} = \frac{1}{6}.$$

Therefore $c = \frac{1}{6}$. (Check: $f(2.001) \approx 0.16666$.)

16. **Answer:** C. $[1,\ 2]$

Because f is continuous, the Intermediate Value Theorem guarantees a zero on an interval exactly when the endpoint values have opposite signs. Evaluate $f(x) = e^{x/2} - x^2$ at each endpoint with a calculator: $f(-2) = e^{-1} - 4 \approx 0.368 - 4 = -3.632$ $f(-1) = e^{-0.5} - 1 \approx 0.607 - 1 = -0.393$ $f(0) = e^0 - 0 = 1$
$f(1) = e^{0.5} - 1 \approx 1.649 - 1 = 0.649$ $f(2) = e^1 - 4 \approx 2.718 - 4 = -1.282$
$f(3) = e^{1.5} - 9 \approx 4.482 - 9 = -4.518$ Now compare signs on each interval: on $[-2,\ -1]$ both values are negative; on $[0,\ 1]$ both are positive; on $[2,\ 3]$ both are negative. Only on $[1,\ 2]$ do the signs differ, since $f(1) \approx 0.649 > 0$ and $f(2) \approx -1.282 < 0$. Since 0 lies between $f(1)$ and $f(2)$ and f is continuous, the theorem guarantees a c in $(1,\ 2)$ with $f(c) = 0$.

17. **Answer:** D. 2.197

Putting $x = 0$ in directly gives $\frac{3^0 - 3^0}{0} = \frac{0}{0}$, an indeterminate form, so estimate the limit numerically instead. Evaluate f on both sides of 0:

x	-0.01	-0.001	0.001	0.01
$f(x)$	2.1977	2.1972	2.1972	2.1977

The values approach the same number from both sides, so $\lim_{x\to 0} f(x) \approx 2.197$. The exact value is $2\ln 3 = 2.19722\ldots$: each of $\frac{3^x - 1}{x}$ and $\frac{1 - 3^{-x}}{x}$ approaches $\ln 3$, and the two contributions add rather than cancel.

18. **Answer:** $5/6$

At $x = 3$ the denominator $x^2 - 9 = (x-3)(x+3)$ vanishes. A finite limit is only possible if the numerator vanishes there as well, which gives

$$3^2 + 3k - 6 = 0 \implies 3 + 3k = 0 \implies k = -1.$$

With $k = -1$ the numerator factors as $x^2 - x - 6 = (x-3)(x+2)$, so for $x \neq 3$

$$g(x) = \frac{(x-3)(x+2)}{(x-3)(x+3)} = \frac{x+2}{x+3}.$$

Therefore $\lim_{x\to 3} g(x) = \frac{3+2}{3+3} = \frac{5}{6}$.

19. **Answer:** C. 4.399

Continuity at $x = 2$ requires the left-hand limit, the right-hand limit, and $f(2)$ to agree. From the left, factor and cancel:

$$\lim_{x\to 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4.$$

From the right, $\lim_{x\to 2^+} b\sin x = b\sin 2 = f(2)$. Setting the two equal: $b\sin 2 = 4$, so

$$b = \frac{4}{\sin 2} = \frac{4}{0.909297} = 4.399.$$

Radian mode matters here: $\sin 2 \approx 0.909$, while $\sin 2^\circ \approx 0.0349$.

20. **Answer:** C. 4

Apply the Intermediate Value Theorem to each subinterval whose endpoint values lie on opposite sides of 2, and also read the table for values equal to 2. $[0, 2]$: f goes from -4 to 3, so $f(c) = 2$ for some c in $(0, 2)$. $[2, 5]$: f goes from 3 to 1, so $f(c) = 2$ for some c in $(2, 5)$. $[5, 7]$: f goes from 1 to 6, so $f(c) = 2$ for some c in $(5, 7)$. $x = 9$: the table gives $f(9) = 2$ outright, a fourth solution. On $[7, 9]$ $(6 \to 2)$ and on $[9, 12]$ $(2 \to -3)$ the theorem guarantees a solution in the closed interval, but $x = 9$ already serves as that solution, so neither forces a new one. A continuous function that is monotonic between consecutive table entries has these four solutions and no others, so four is the least number that must exist.

21. **Answer:** D. $\dfrac{3}{8}$

At $x = -3$ the expression reads $\frac{\sqrt{16}-4}{0} = \frac{0}{0}$, so rationalize the numerator. Multiply above and below by the conjugate $\sqrt{x^2+7} + 4$:

$$\frac{\sqrt{x^2+7}-4}{x^2+4x+3} \cdot \frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4} = \frac{(x^2+7)-16}{(x+1)(x+3)\left(\sqrt{x^2+7}+4\right)} = \frac{x^2-9}{(x+1)(x+3)\left(\sqrt{x^2+7}+4\right)}.$$

Factor $x^2 - 9 = (x-3)(x+3)$ and cancel the common factor $x + 3$:

$$\frac{x-3}{(x+1)\left(\sqrt{x^2+7}+4\right)}.$$

Now substitute $x = -3$:

$$\frac{-6}{(-2)(4+4)} = \frac{-6}{-16} = \frac{3}{8}.$$

22. **Answer:** D. 12

Both bounding functions approach the same value at $x = 0$: $\lim_{x\to 0}(4 - x^2) = 4$ and $\lim_{x\to 0}(4 + x^2) = 4$. By the Squeeze Theorem, $\lim_{x\to 0} f(x) = 4$. For the second factor, rewrite so the special limit $\lim_{u\to 0} \frac{\sin u}{u} = 1$ applies:

$$\frac{\sin 3x}{x} = 3 \cdot \frac{\sin 3x}{3x} \;\longrightarrow\; 3 \cdot 1 = 3.$$

Both limits exist, so the limit of the product is the product of the limits:

$$\lim_{x\to 0}\left(f(x) \cdot \frac{\sin 3x}{x}\right) = 4 \cdot 3 = 12.$$