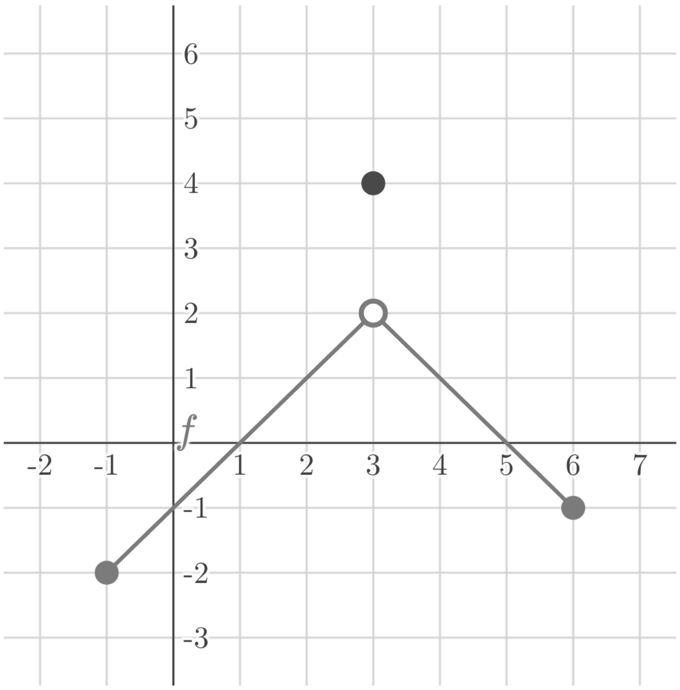
Limits Practice Packet — Graphical & Algebraic

Name ______________________ Date ____________________ *AP Calculus · 1729math.com*

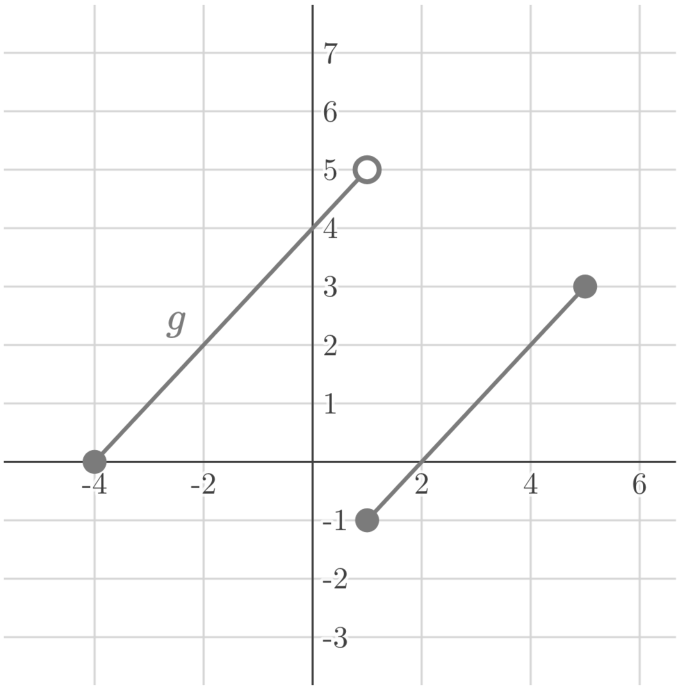
1. The graph of $y = f(x)$ is shown. The open circle at $(3, 2)$ is a hole in the graph, and the filled dot shows the value of $f(3)$. Use the graph to fill in each value.



$\lim_{x\to 3} f(x)$: ______________________

$f(3)$: ______________________

2. The graph of $y = g(x)$ is shown. Use the graph to evaluate each expression. If a limit does not exist, write DNE.

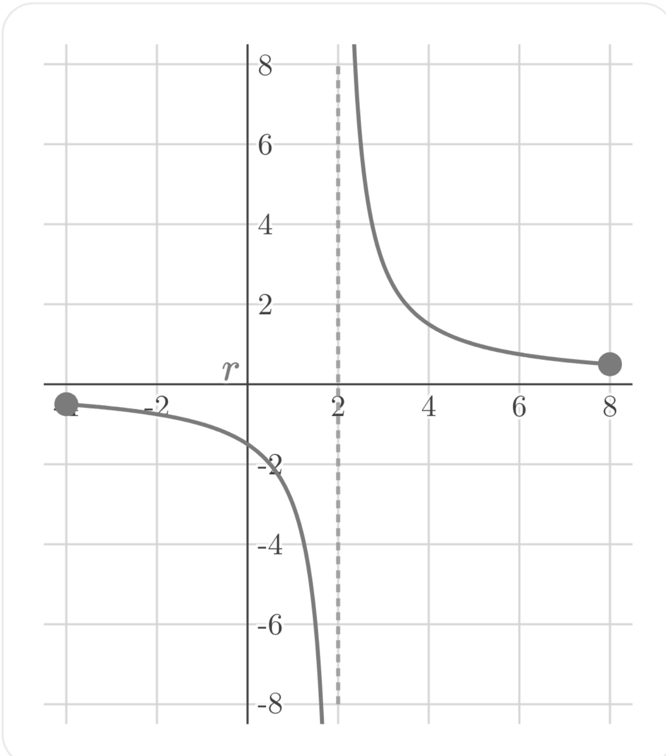


$\lim_{x \to 1^-} g(x)$: ______________________________

$\lim_{x \to 1^+} g(x)$: ______________________________

$\lim_{x \to 1} g(x)$: ______________________________

3. The graph of $y = r(x)$ is shown; the dashed line $x = 2$ is a vertical asymptote. Evaluate $\lim_{x \to 2^+} r(x)$.



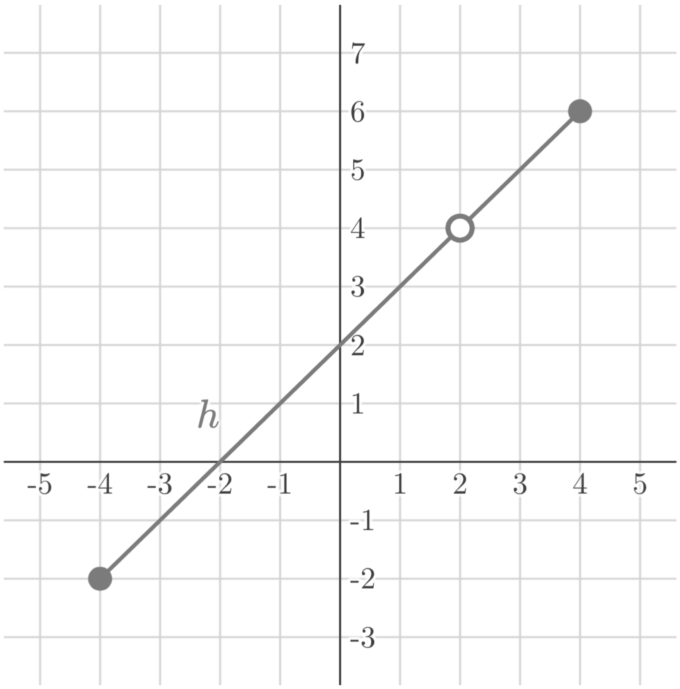
A) $-\infty$

B) ∞

C) 0

D) 3

4. The graph of $y = h(x)$ is shown. It is a line with a single hole: $h(2)$ is undefined. What value should be assigned to $h(2)$ to make h continuous at $x = 2$?



$h(2)$: ____________________

5. Evaluate: $\lim_{x \to 2} \left(x^3 - 4x + 1\right)$

____

6. Evaluate: $\lim_{x \to 5} \dfrac{x^2 - 25}{x - 5}$

____

7. Evaluate: $\lim_{x \to -2} \dfrac{x^2 + 5x + 6}{x^2 - 4}$

____

8. Evaluate: $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

____

9. Evaluate: $\lim_{h \to 0} \frac{\frac{1}{4+h} - \frac{1}{4}}{h}$

____

10. The function p is defined by

$$p(x) = \begin{cases} x^2 + 1 & \text{if } x < 2 \\ 7 - x & \text{if } x \geq 2 \end{cases}$$

Evaluate each limit.

$\lim_{x \to 2^-} p(x)$: ___________________________

$\lim_{x \to 2^+} p(x)$: ___________________________

$\lim_{x \to 2} p(x)$: ___________________________

11. Evaluate: $\lim_{x \to 0} \frac{\sin(4x)}{x}$

____

12. Evaluate: $\lim_{x \to \infty} \frac{8x^2 - 3x + 1}{2x^2 + 5x}$

A) 0

B) ∞

C) 4

D) 8

Limits Practice Packet — Graphical & Algebraic — Answer Key

Answer key with worked solutions — *AP Calculus · 1729math.com*

1. **Answer:** $\lim_{x\to 3} f(x)$: 2, $f(3)$: 4

   As x approaches 3 from the left, the graph climbs toward a height of 2; from the right it also heads toward height 2. Both sides agree, so $\lim_{x\to 3} f(x) = 2$. The filled dot at $(3, 4)$ gives the actual function value: $f(3) = 4$. A limit describes the height the graph is heading toward, not the value plotted there — the two need not match.

2. **Answer:** $\lim_{x\to 1^-} g(x)$: 5, $\lim_{x\to 1^+} g(x)$: -1, $\lim_{x\to 1} g(x)$: DNE

   From the left, the graph rises along the line $y = x + 4$ toward the open circle at $(1, 5)$, so $\lim_{x\to 1^-} g(x) = 5$. From the right, the graph follows $y = x - 2$ and approaches the filled dot at $(1, -1)$, so $\lim_{x\to 1^+} g(x) = -1$. The one-sided limits disagree $(5 \neq -1)$, so $\lim_{x\to 1} g(x)$ does not exist (DNE).

3. **Answer:** B. ∞

   Just to the right of $x = 2$, the graph shoots upward along the asymptote: the closer x gets to 2 from the right, the larger $r(x)$ grows without bound. We write $\lim_{x\to 2^+} r(x) = \infty$. From the left the graph plunges downward instead — the two sides behave very differently, which is why the one-sided limit is specified.

4. **Answer:** 4

   Continuity at $x = 2$ requires $h(2) = \lim_{x\to 2} h(x)$. Both branches of the line head toward the hole at $(2, 4)$, so $\lim_{x\to 2} h(x) = 4$. Filling the hole — defining $h(2) = 4$ — makes the function continuous there. This is called a removable discontinuity.

5. **Answer:** 1

   Polynomials are continuous everywhere, so the limit is found by direct substitution:
   $2^3 - 4(2) + 1 = 8 - 8 + 1 = 1$.

6. **Answer:** 10

   Substituting $x = 5$ gives $\frac{0}{0}$, an indeterminate form — so simplify first. Factor the numerator: $\frac{(x-5)(x+5)}{x-5} = x + 5$ for $x \neq 5$. The limit of $x + 5$ as $x \to 5$ is 10.

7. **Answer:** -1/4

   Substituting $x = -2$ gives $\frac{0}{0}$. Factor both quadratics: $\frac{(x+2)(x+3)}{(x+2)(x-2)} = \frac{x+3}{x-2}$ for $x \neq -2$. Now substitute: $\frac{-2+3}{-2-2} = \frac{1}{-4} = -\frac{1}{4}$.

8. **Answer:** 1/6

   Substitution gives $\frac{0}{0}$. Multiply numerator and denominator by the conjugate $\sqrt{x} + 3$: $\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3}$. Substituting $x = 9$: $\frac{1}{3+3} = \frac{1}{6}$.

9. **Answer:** -1/16

   Combine the fractions in the numerator: $\frac{1}{4+h} - \frac{1}{4} = \frac{4-(4+h)}{4(4+h)} = \frac{-h}{4(4+h)}$. Dividing by h cancels it, leaving $\frac{-1}{4(4+h)}$. As $h \to 0$ this approaches $-\frac{1}{16}$. This limit is the derivative of $\frac{1}{x}$ at $x = 4$ — a preview of what comes next.

10. **Answer:** $\lim_{x \to 2^-} p(x)$: 5, $\lim_{x \to 2^+} p(x)$: 5, $\lim_{x \to 2} p(x)$: 5

From the left ($x < 2$), use the first rule: $\lim_{x \to 2^-} p(x) = 2^2 + 1 = 5$. From the right ($x \geq 2$), use the second rule: $\lim_{x \to 2^+} p(x) = 7 - 2 = 5$. The one-sided limits agree, so the two-sided limit exists: $\lim_{x \to 2} p(x) = 5$. Different formulas on the two sides do not automatically break a limit — what matters is whether both sides approach the same value.

11. **Answer:** 4

Use the special limit $\lim_{u \to 0} \frac{\sin u}{u} = 1$. Rewrite so the arguments match: $\frac{\sin(4x)}{x} = 4 \cdot \frac{\sin(4x)}{4x}$. As $x \to 0$, $4x \to 0$, so $\frac{\sin(4x)}{4x} \to 1$ and the limit is $4 \cdot 1 = 4$.

12. **Answer:** C. 4

For end behavior of a rational function, the highest-degree terms dominate. Numerator and denominator both have degree 2, so the limit is the ratio of the leading coefficients: $\frac{8}{2} = 4$. Formally: divide every term by x^2; the terms $\frac{3}{x}$, $\frac{1}{x^2}$, and $\frac{5}{x}$ all go to 0, leaving $\frac{8}{2}$.